

DSP: Lec 2

Given $x(n) = \left\{ \underset{\substack{\downarrow \\ -2}}{1}, \underset{\substack{\uparrow \\ -1}}{\frac{1}{2}}, \underset{\substack{\uparrow \\ 0}}{1}, \underset{\substack{\uparrow \\ 1}}{0}, \underset{\substack{\uparrow \\ 2}}{-1}, \underset{\substack{\uparrow \\ 3}}{\frac{1}{2}} \right\}$

write
a) equation in terms of unit impulse

b) sketch $x(n)$ c) find $x(-n+2)$

Sol

$$a) x(n) = \delta(n+2) + \frac{1}{2} \delta(n+1) + \delta(n) - \delta(n-2) + \frac{1}{2} \delta(n-3)$$

b/c \rightarrow From the previous Lec.

classification of discrete-time systems:-

[1] static and dynamic discrete-time sys.

static \Rightarrow o/p of the system depends
(memoryless)
up on the present i/p only.

dynamic \Rightarrow o/p depends up on the past -
(memory) i/p also.

[1] Lec 2

[Ex] * $y(n) = 10 x(n) \rightarrow \text{static} \Rightarrow \text{Causal}$

* $y(n) = 15 x^2(n) + \cos(n) \rightarrow \text{static} \Rightarrow \text{Causal}$

* $y(n) = x(n-1) + 3 x(n)^2 \rightarrow \text{dynamic} \Rightarrow \text{Causal}$

* $y(n) = x(n+1) + x(n) + x(n-1) \rightarrow \text{dynamic} \Rightarrow \text{non-Causal}$

\rightarrow if system need memory to store values in input \rightarrow dynamic (memory)

[2] Causal and non Causal system:-

* Causal system \Rightarrow o/p depends on the Present & Past i/p :

* non Causal system \Rightarrow o/p depends on the Future i/p also.

← الوحدة الأساسية في الـ (non Causal) إنه الخرج يعتمد على القيمة المستقبلية للدخل لكنه بالإضافة أنه أيضاً يعتمد على قيم الماضي والحاضر .

→ non Causal sys \Rightarrow (un Reliable)

$$* y(n) = x(2n)$$

$$n=0 \Rightarrow y(0) = x(0)$$

$$n=1 \Rightarrow y(1) = x(2) \rightarrow \text{non Causal}$$

$$* y(n) = x(n^2) \rightarrow \text{non Causal}$$

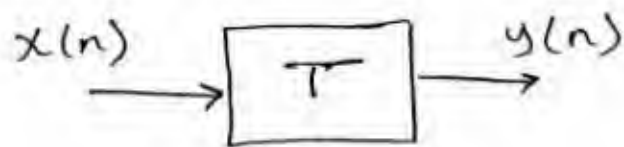
[3] shift variant system - shift invariant system
(time variant sys. - time invariant sys.)

* time invariant system \Rightarrow o/p ch/s don't
change with shifting in time.

* time variant system \Rightarrow o/p ch/s change
with shifting in time.

[3] Lec 2

→ For discrete time system:



$$y(n) = T[x(n)] \quad \leftarrow \text{system operation}$$

$$y(n, k) = T[x(n-k)] \quad \text{shift operation}$$

$$y(n-k) = y(n) \Big|_{n \rightarrow n-k}$$

→ if $y(n, k) = y(n-k) \Rightarrow$ time invariant system.

→ if $y(n, k) \neq y(n-k) \Rightarrow$ time variant sys.

Ex 1- $y(n) = x(n) \cos(\omega n)$

$$y(n, k) = x(n-k) \cos(\omega n)$$

$$y(n-k) = y(n) \Big|_{n \rightarrow n-k} = x(n-k) \cos(\omega n-k)$$

$y(n, k) \neq y(n-k) \Rightarrow$ time variant system.

$$2- y(n) = x(n) + x(n-2)$$

$$y(n, K) = x(n-K) + x(n-K-2)$$

$$y(n-K) = y(n) \Big|_{n \rightarrow n-K} = x(n-K) + x(n-K-2)$$

$$y(n-K) = y(n, K) \rightarrow \text{time invariant system.}$$

$$3- y(n) = x(-n)$$

$$y(n, K) = x(-n-K)$$

$$y(n-K) = y(n) \Big|_{n \rightarrow n-K} = x(-(n-K)) = x(-n+K)$$

$$y(n-K) \neq y(n, K) \rightarrow \text{time variant system.}$$

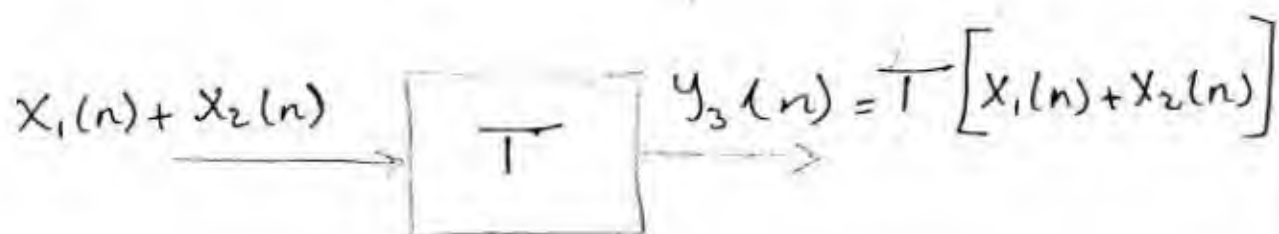
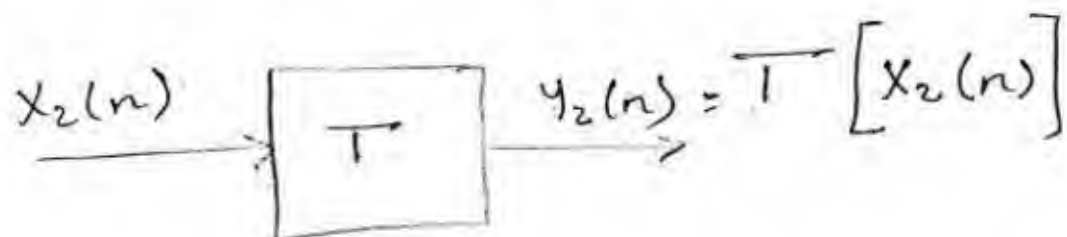
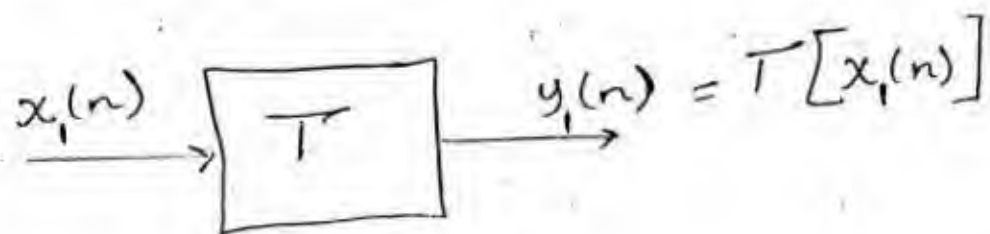
$$\text{Ex: } y(n) = x(n^2)$$

$$y(n, K) = x(n^2 - K)$$

$$y(n-K) = y(n) \Big|_{n \rightarrow n-K} = x((n-K)^2)$$

$$y(n, K) \neq y(n-K) \rightarrow \text{time variant system.}$$

[4] Linear and non-Linear system:-



if: $y_3(n) = y_1(n) + y_2(n) \Rightarrow$ Linear system.

if: $y_3(n) \neq y_1(n) + y_2(n) \Rightarrow$ non-Linear system.

[Ex] $y(n) = x(n^2)$

$$y_1(n) = x_1(n^2) \quad \& \quad y_2(n) = x_2(n^2)$$

$$y_3(n) = x_1(n^2) + x_2(n^2)$$

$$\therefore y_3(n) = y_1(n) + y_2(n) \rightarrow \text{Linear sys.}$$

$$2. \underline{y(n) = x^2(n)}$$

$$y_1(n) = x_1^2(n)$$

$$y_2(n) = x_2^2(n)$$

$$y_3(n) = [x_1(n) + x_2(n)]^2$$

$$y_3(n) \neq y_1(n) + y_2(n) \rightarrow \text{non-Linear sys.}$$

$$3. y(n) = \log [x(n)]$$

\rightarrow non-Linear system.

$$4. y(n) = \cos [x(n)] \rightarrow \text{non-Linear}$$

[5] stable and unstable system:-

stable \Rightarrow For ^{every} x bounded i/p sequence,
there exist bounded o/p \therefore ~~B~~

$[B \leq B_0] \rightarrow$ bounded i/p bounded o/p

[7] Lec 2

Ex

1) $y(n) = x(n^2) \rightarrow \text{stable}$

2) $y(n) = x^2(n) \rightarrow \text{stable}$

3) $y(n) = 2^n x(n) \rightarrow \text{unstable}$

4) $y(n) = x(n) \cos(\omega n) \rightarrow \text{stable}$

5) $y(n) = n x(n) \rightarrow \text{unstable}$

Ex $y(n) = x(-n+2)$

1. Dynamic

2. non-causal (depend on future) $y(0) = x(2)$

3. $y(n, k) = x(-n+2-k)$

$$y(n-k) = y(n) \big|_{n \rightarrow n-k} = x(-(n-k)+2) \\ = x(-n+k+2)$$

\rightarrow time variant system.

4. $y_1(n) = x_1(-n+2)$, $y_2(n) = x_2(-n+2)$

$y_3(n) = x_1(-n+2) + x_2(-n+2) = y_1(n) + y_2(n)$

\rightarrow Linear

8 Lec 2

Ex

$$y(n) = x(n) + n x(n-1)$$

5) stable-system.

EX $y(n) = x(n) + n x(n+1)$

1) Dynamic 2) non-causal

$$3) y(n, k) = x(n-k) + n x(n+1-k)$$

$$y(n-k) = y(n) \Big|_{n \rightarrow n-k} \quad \text{ ~~$x(n-k)$~~ }$$

$$= x(n-k) + (n-k) x(n+1-k)$$

$$y(n, k) \neq y(n-k) -$$

→ time-variant

4) Linear

5) un-stable.

[9] Lec 2

$$y(n) = \cos[x(n)]$$

- 1) static
 - 2) causal.
 - 3) time-invariant.
 - 4) non-linear.
 - 5) stable.
-

→ Report check the five properties.

$$1) y(n) = \sum_{k=-a}^{n+1} x(k)$$

$$2) y(n) = x(2n)$$

$$3) y(n) = x(-n)$$

[10] Lec 2

→ Linear Convolution:-

$$x(n) = \sum_{k=-\infty}^{\infty} x(k) \delta(n-k)$$

→ To get $x(n)$

assume $k=n$

$$x(n) = \sum_{-\infty}^{\infty} x(n)$$

→ any discrete time signal can be expressed in terms of unit sample sequence (unit impulse).

→ For a discrete time system:-

$$x(n) \rightarrow \boxed{T} \rightarrow y(n) = T[x(n)]$$

$$y(n) = T[x(n)]$$

$$= T \left[\sum_{k=-\infty}^{\infty} x(k) \delta(n-k) \right]$$

□ 11 □ Lec 2

→ The system is assumed to be
Linear time invariant system (LTI)

$$y(n) = T \left[\dots + \{x(-2) \delta(n+2) + \right. \\ \left. x(-1) \delta(n+1) + x(0) \delta(n) + x(1) \delta(n-1) \right. \\ \left. + x(2) \delta(n-2) + \dots \right]$$

as the system is Linear:-

$$y(n) = \dots + T [x(-2) \delta(n+2)] + \\ T [x(-1) \delta(n+1)] + T [x(0) \delta(n)] + \\ + T [x(1) \delta(n-1)] + T [x(2) \delta(n-2)] \\ + \dots$$

~~$x(-2), x(-1), x(2)$~~

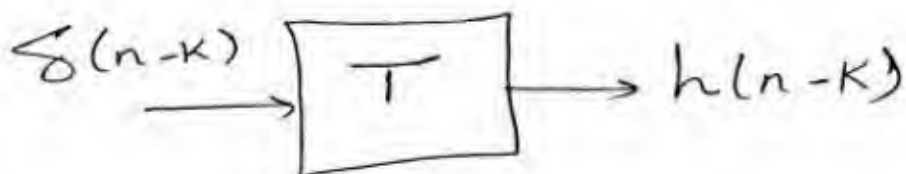
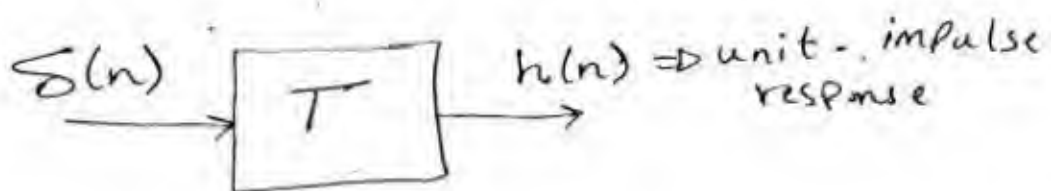
$\dots, x(-1), x(0), x(1), \dots$ Constant values

$$y(n) = \dots + x(-2) T[\delta(n+2)] \\ + x(-1) T[\delta(n+1)] + x(0) T[\delta(n)] \\ + x(1) T[\delta(n-1)] + x(2) T[\delta(n-2)] \dots$$

~~$$y(n) = \sum_{k=-\infty}^{\infty} x(k) T[\delta(n-k)]$$~~

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) T[\delta(n-k)]$$

assume



$$h(n) = T[\delta(n)]$$

$$h(n-k) = T[\delta(n-k)]$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

Linear Convolution

~~input signal~~

→ The system response to any i/p sequence $x(n)$, can be calculated by the known of unit-impulse response $h(n)$.

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

$x(n) \rightarrow$ the i/p sequence.

$y(n) \rightarrow$ the required response for the i/p $x(n)$

$h(n) \rightarrow$ the known unit impulse response

$$h(n) = y(n) \mid x(n) = \delta(n)$$

[EX] $x(n) = \{ \underset{\substack{\uparrow \\ n=0}}{1}, \underset{\substack{\uparrow \\ n=3}}{1}, 1, 1 \}$, $h(n) = \{ \underset{\substack{\uparrow \\ n=0}}{2}, \underset{\substack{\uparrow \\ n=1}}{2} \}$

* Compute the Convolution of the two sequences.

Sol

$$y(n) = \sum_{K=-\infty}^{\infty} x(K) h(n-K)$$

$$n_{\text{start}} = n_{x_{\text{start}}} + n_{h_{\text{start}}} = 0 + 0 = 0$$

$$n_{\text{end}} = n_{x_{\text{end}}} + n_{h_{\text{end}}} = 3 + 1 = 4$$

$4 \leftarrow 0$ as n is signed \leftarrow

$$n=0 \Rightarrow y(0) = \sum_{K=-\infty}^{\infty} x(K) h(-K)$$

$$= \sum_{K=0}^3 x(K) h(-K)$$

[15] Lec 2

$$y(0) = x(0)h(0) + x(1)h(-1) \\ + x(2)h(-2) + x(3)h(-3)$$

$$y(0) = x(0)h(0) = 1 \times 2 = \boxed{2}$$

$$n=1 \Rightarrow y(1) = \sum_{k=0}^3 x(k)h(1-k)$$

$$= x(0)h(1) + x(1)h(0) + x(2)h(-1) \\ + x(3)h(-2)$$

$$= 1 \times 2 + 2 \times 1 = 4$$

$$n=2 \Rightarrow y(2) = \sum_{k=0}^3 x(k)h(2-k)$$

$$= x(0)h(2) + x(1)h(1) + x(2)h(0) \\ + x(3)h(-1)$$

$$= 2 \times 1 + 1 \times 2 = 4$$

$\boxed{16}$ Lec 2

$$n=3 \Rightarrow y(3)=4$$

$$n=4 \Rightarrow y(4)=2$$

$$y(n) = \{ \underset{\uparrow}{2}, 4, 4, \cancel{4}, 2 \}$$

$$\rightarrow x(n) = \{ 1, 1, \underset{\substack{\uparrow \\ n=0}}{0}, 1, 1 \}$$

$$h(n) = \{ 1, -2, -3, \underset{\substack{\uparrow \\ n=0}}{4} \}$$

Sol

$$y(n) = \sum_{K=-\infty}^{\infty} x(K) h(n-K)$$

$$n_{\text{start}} = n_{x \text{ start}} + n_{h \text{ start}}$$

$$= -2 - 3 = -5$$

$$n_{\text{end}} = n_{x \text{ end}} + n_{h \text{ end}}$$

$$= 2 + 0 = 2$$

Ans P

~~Handwritten scribbles at the top of the page.~~

$$\underline{n = -5} \quad y(-5) = \sum_{K=-2}^2 x(K) h(n-K)$$

$$y(-5) = \sum_{-2}^2 x(K) h(-5-K)$$

$$= x(-2) h(-3) + x(-1) h(-4) + x(0) h(-5) + x(1) h(-6) + x(2) h(-7)$$

$$= x(-2) h(-3) = 1 * 1 = 1$$

$$\boxed{n = -4} \quad y(-4) = \sum_{K=-2}^2 x(K) h(-4-K)$$

$$= x(-2) h(-2) + x(-1) h(-3) + x(0) h(-4) + x(1) h(-5) + x(2) h(-6)$$

$$y(-4) = -2 + 1 = \boxed{-1}$$

$$\underline{n = -3}$$

$$y(-3) = -5$$

$$\underline{n = -2}$$

$$y(-2) = 2$$

$$\underline{n = -1}$$

$$y(-1) = 3$$

$$\underline{n = 0} \Rightarrow y(0) = -5$$

$$\underline{n = 1} \Rightarrow y(1) = 1$$

$$\underline{n = 2} \Rightarrow y(2) = 4$$

$$y(n) = \{1, -1, -5, 2, 3, -5, 1, 4\}$$

$$y(n) = x(n) * h(n)$$

\searrow Convolution

$$= \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

[19] Lec 2